Free-fall and LISA sensitivity below 0.1 mHz

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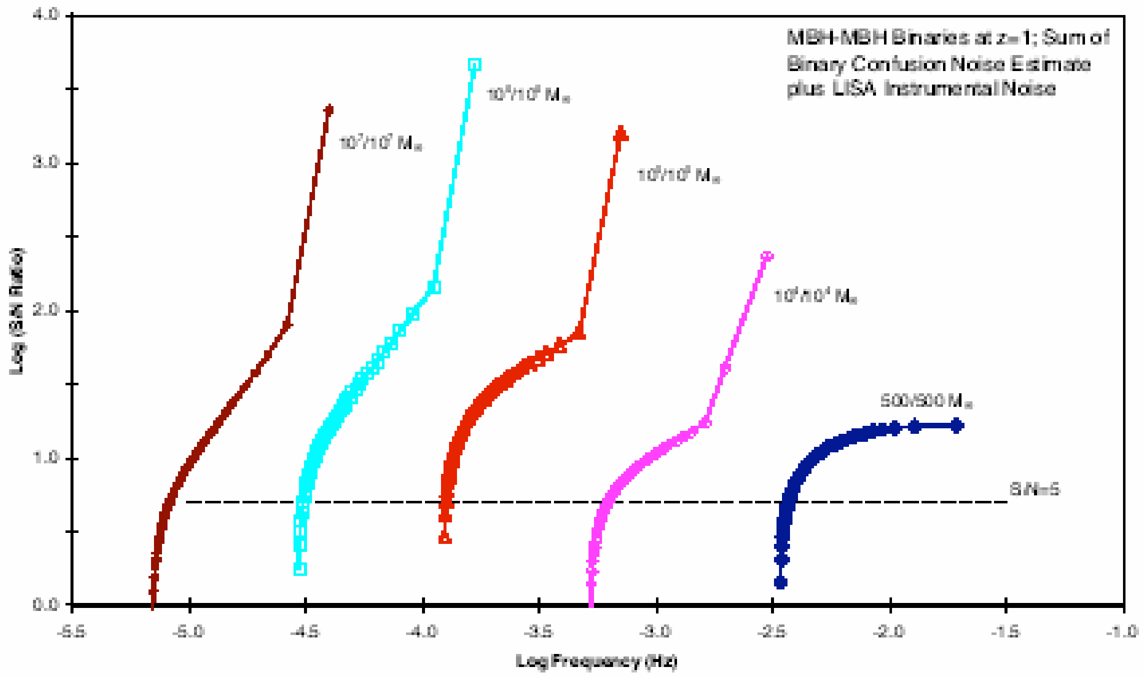
Purity of free-fall critical to LISA science

Example: massive black hole (MBH) mergers
Integrated SNR at 1 week intervals for year before merger



Assuming LISA goal:

$S_a^{1/2} < 3$ fm/s^2/Hz$^{1/2}$
at 0.1 mHz

Acceleration noise at and below 0.1 mHz determines how well, how far, and how early we will see the most massive black hole mergers.

Key acceleration noise sources for f < 0.1 mHz

- Interactions with TM magnetic moment
- Thermal gradient induced-forces

  → *see talks by Pete Bender, Mauro Hueller and Scott Pollack*
- Noisy low frequency electrostatics

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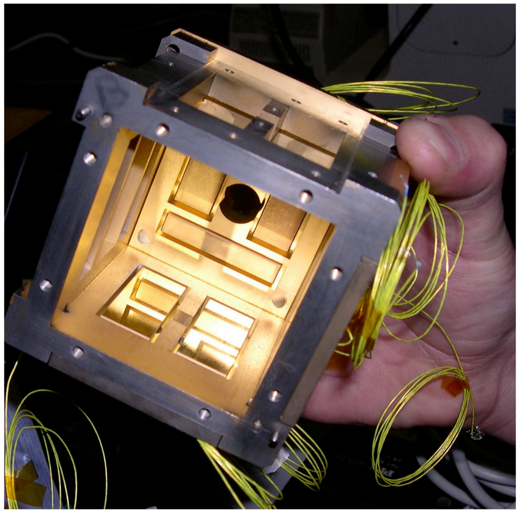
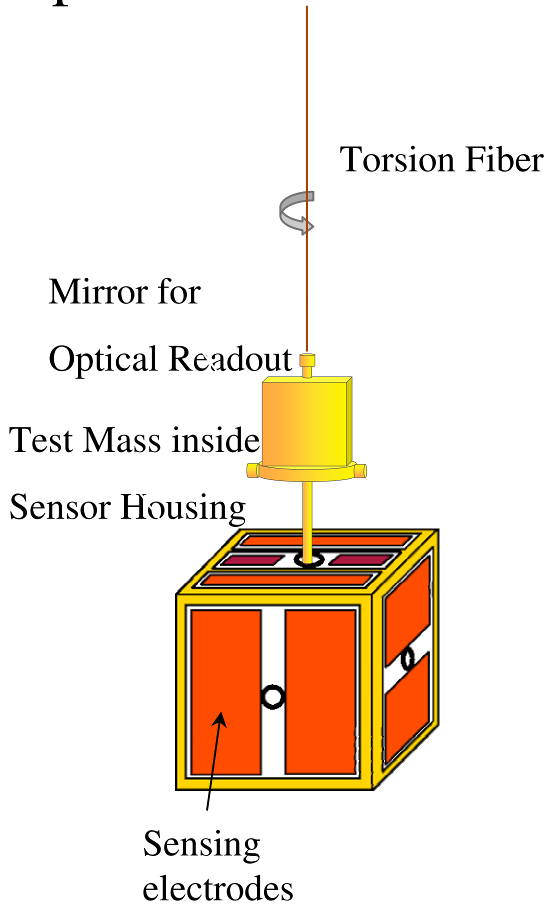
- **Noisy low frequency electrostatics**



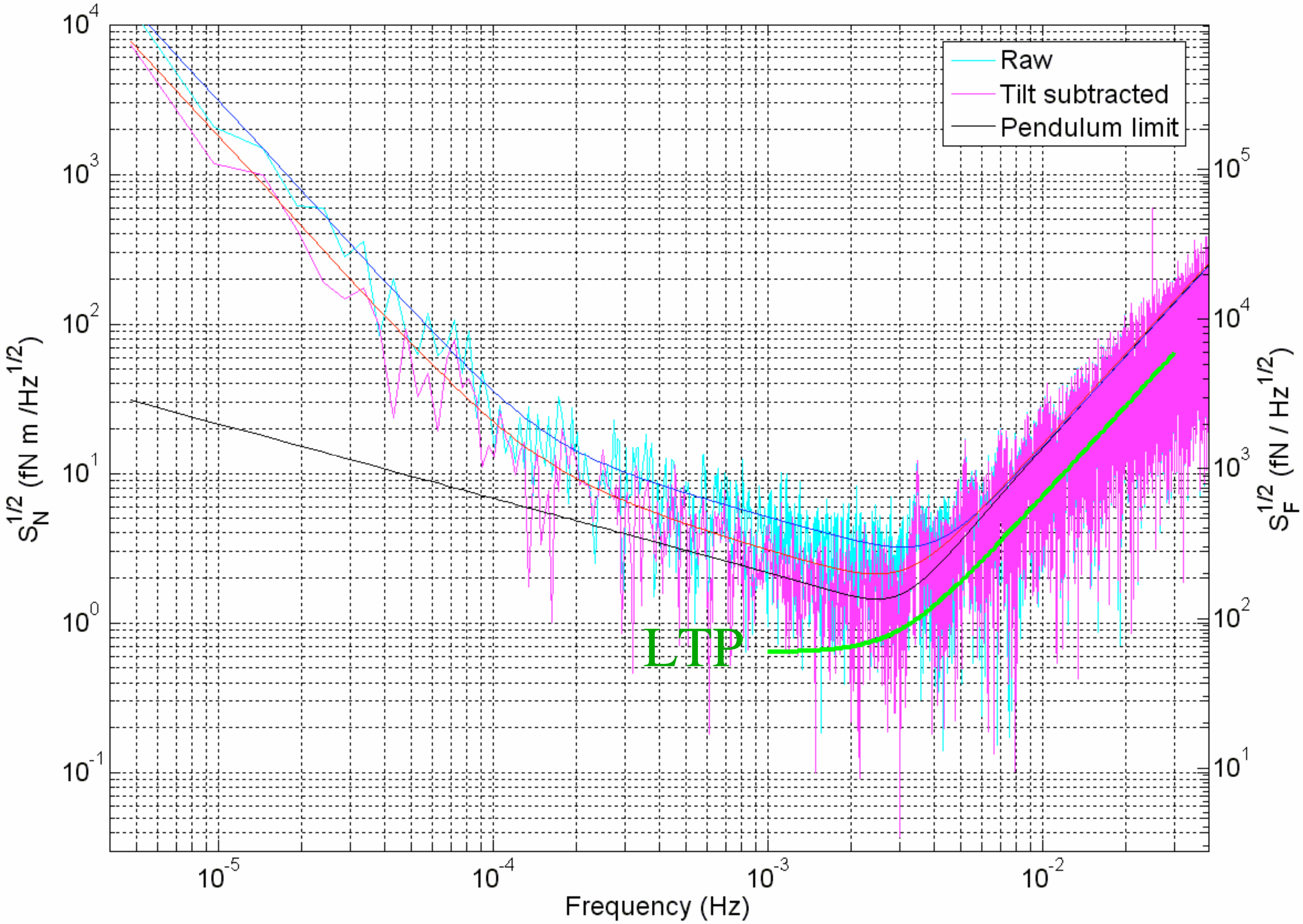
Measurements performed with Engineering Model sensor for LTP

- 4 mm x-sensing gaps
- Mo / Au-coated Shapal

1-mass torsion pendulum

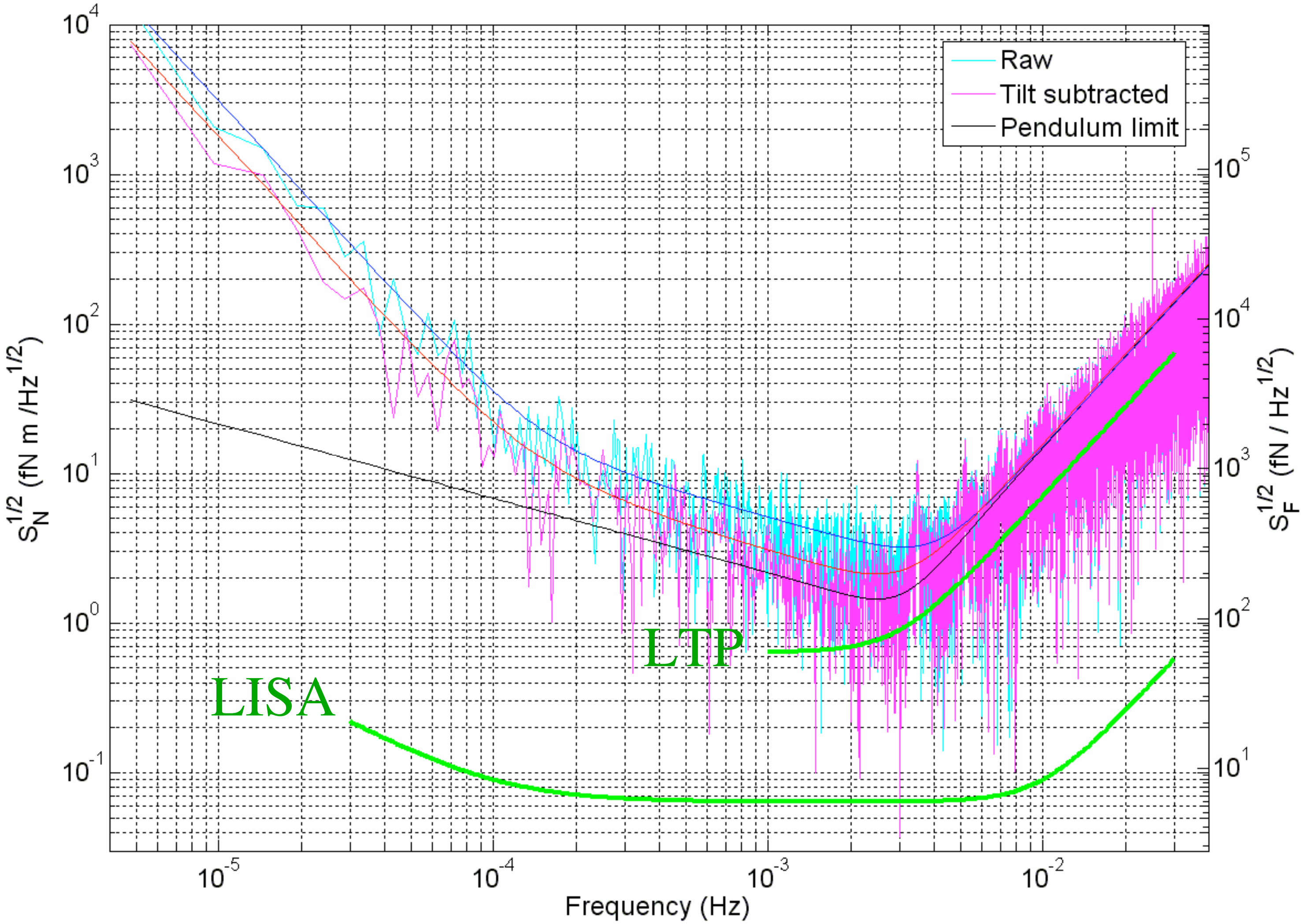


Sensor force noise upper limits from torsion pendulum noise data



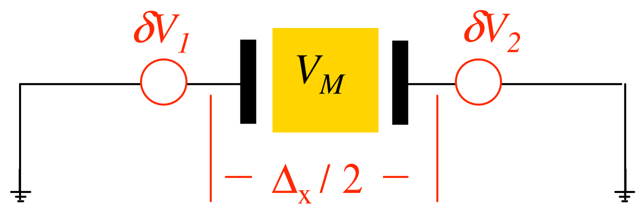
- Factor of 3 above LTP goal at 2 mHz

Sensor force noise upper limits from torsion pendulum noise data



- Factor of 50 above LISA goal at 1 mHz
- Factor of 300 above LISA goal at 0.1 mHz

Noise source: stray low frequency electrostatics



$$k \equiv -\frac{\partial F}{\partial x} = -\frac{1}{2}\sum \frac{\partial^2 C_i}{\partial x^2}(\delta V_i)^2$$

Electrostatic stiffness

$$F = \frac{Q}{C_{TOT}}\sum \frac{\partial C_i}{\partial x}\delta V_i \quad \left\{ \begin{array}{l} S_F^{1/2} = \dfrac{\sqrt{2e^2\lambda_{EFF}}}{\omega C_T}\left|\dfrac{\partial C}{\partial x}\right|\Delta_x \\[2ex] S_F^{1/2} = \dfrac{\langle Q\rangle}{C_T}\left|\dfrac{\partial C}{\partial x}\right| S_{\Delta_x}^{1/2} \end{array} \right.$$

Random charge noise mixing with DC bias (Δ_x)

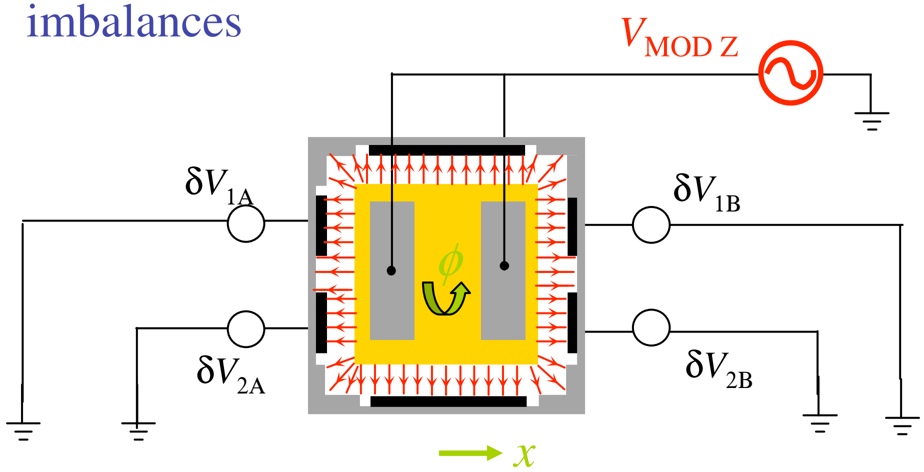
Noisy average "DC" bias ($S_{\Delta x}$) mixing with mean charge

$$S_F^{1/2} = \sqrt{\sum \left|\frac{\partial C_i}{\partial x}\right|^2 \delta V_i^{\,2} S_{\delta V_i}}$$

Noisy "DC" biases interacting with themselves

DC Bias measurement and compensation (in lab and in flight)

- Applied oscillating TM bias simulates TM "charge"
- Excites torque and force proportional to integrated rotational and translational DC bias imbalances



$$N = -V_M \left[\sum \frac{\partial C_i}{\partial \phi} V_i \right] \equiv -V_M \left| \frac{\partial C_x}{\partial \phi} \right| \Delta_\phi$$

$$F = -V_M \left[\sum \frac{\partial C_i}{\partial x} V_i \right] \equiv -V_M \left| \frac{\partial C_x}{\partial x} \right| \Delta_x$$

Δ_ϕ and Δ_x :

- "averaged" rotational and translational DC bias imbalances
- couple directly to TM charge to produce a torque (force)
- With torsion pendulum, measure and compensate Δ_ϕ
- Δ_ϕ statistically similar to translational imbalance Δ_x

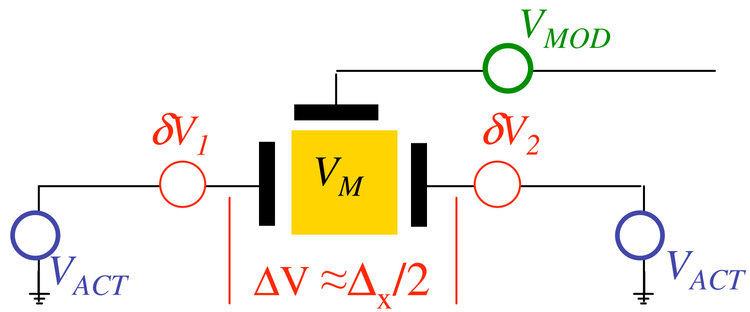
NB: for spatially uniform DC biases: $\Delta_x = \delta V_{1B} + \delta V_{2B} - \delta V_{1B} - \delta V_{2B}$

$\Delta_\phi = -\delta V_{1B} + \delta V_{2B} - \delta V_{1A} + \delta V_{2A}$

Noise source: DC biases and charge shot noise

Fluctuating test mass charge (cosmic ray shot noise)
forced by stray DC electrostatic "patch" fields

$$F = -\frac{C}{d} V_M \;\; \Delta V$$



$$S_a^{1/2}(f) \sim 6\ \mathrm{fm/s^2}/\sqrt{\mathrm{Hz}} \left(\frac{\lambda_{eff}}{800/s}\right)^{1/2} \left(\frac{\Delta_x}{100\ \mathrm{mV}}\right)\left(\frac{10^{-4}\ \mathrm{Hz}}{f}\right)$$

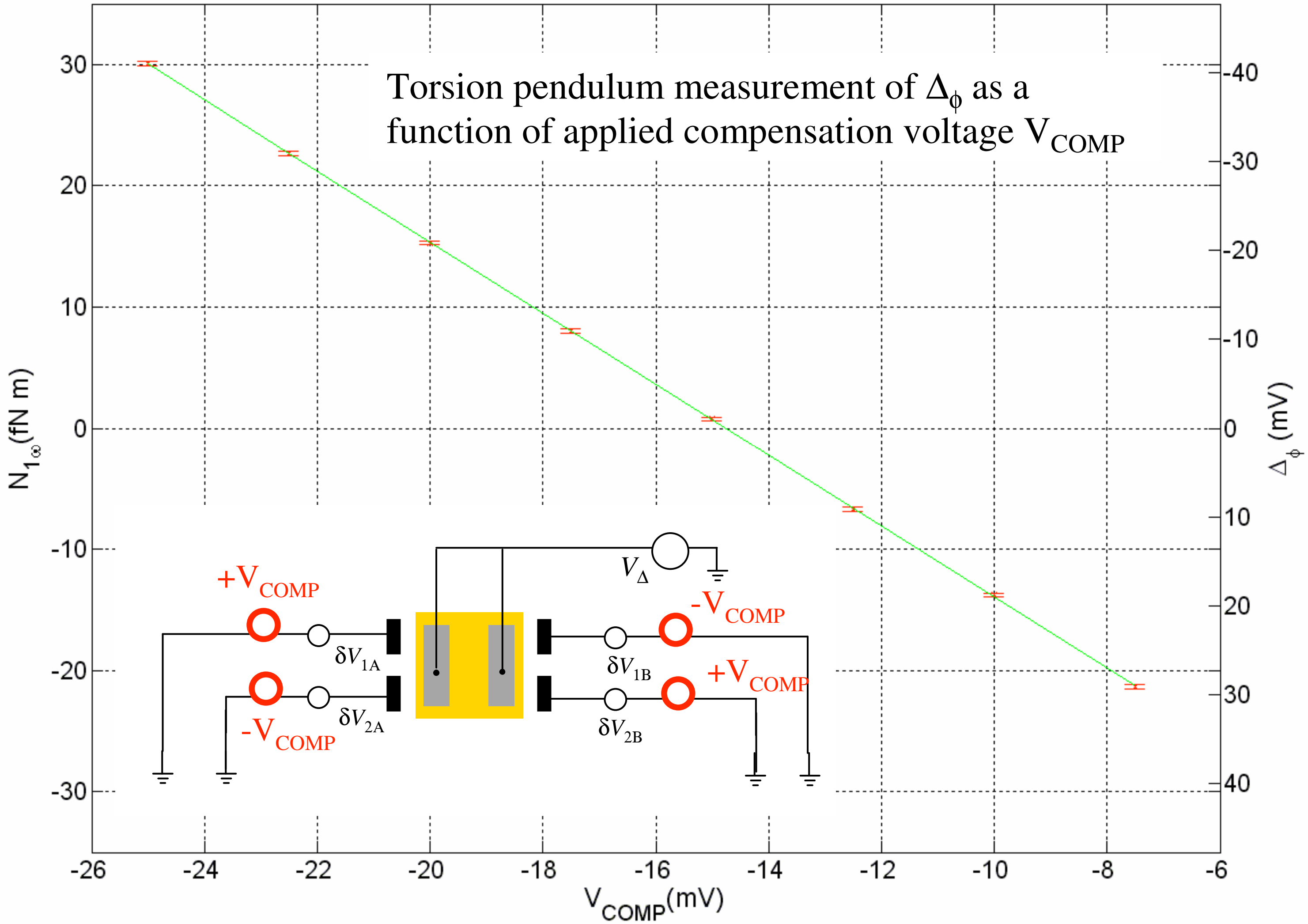
$$\left[\propto \frac{1}{d}\right]$$

- λ_{eff} ~ 800 e/s (H. Araujo, LISA Symposium 2004) includes +/-, different charge number

Charge feels integrated effect from all patch fields

- Can be measured by applying a coherent TM bias (simulated charge)
- Can be cancelled by application of correct compensation voltage

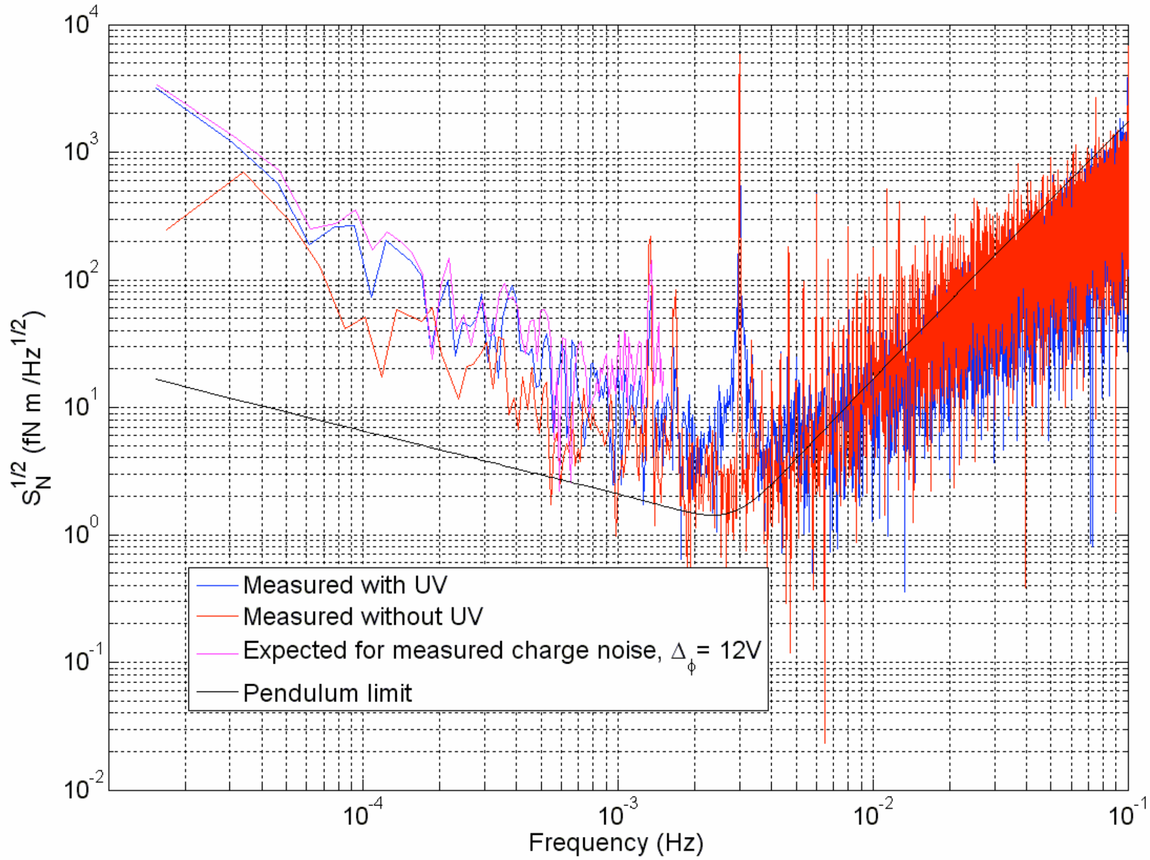
DC Bias: measurement and compensation



- DC biases compensated with V_{COMP} = +15 mV (intrinsic Δ_ϕ = -60 mV)
- Sub-mV measurement possible in 15 minutes integration
- Compensation possible to DAC resolution, in flight
- Random charging should not be problematic under normal conditions

Experimental verification of random charge force noise model

[see talk by Markus Schulte, poster by Imperial / Trento]



Torque noise excess with:

- large TM charge fluctuations produce by UV illumination

$\lambda_{EFF} > 20000$ e/s

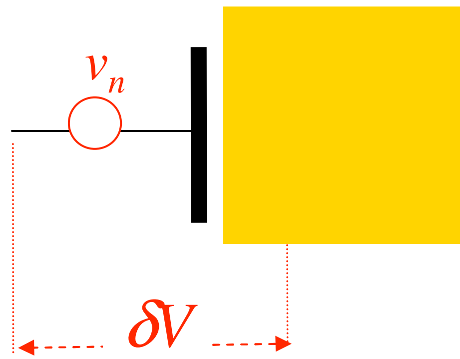
- large applied rotational DC bias

$\Delta_\phi = 12$ V

- Observe low frequency excess in torque noise, in quantitative agreement with random charge model and measured charge fluctuations:

$$N \approx -V_M \left[\sum \frac{\partial C_i}{\partial \phi} V_i \right] \approx -\frac{Q_{TM}}{C_{TOT}} \left| \frac{\partial C_x}{\partial \phi} \right| \Delta_\phi$$

Noise source: in-band voltage noise mixing with DC bias



$$F \approx -\frac{C}{d}\,\delta V \;\; v_{\mathrm{n}}$$

Voltage noise: v_{n}

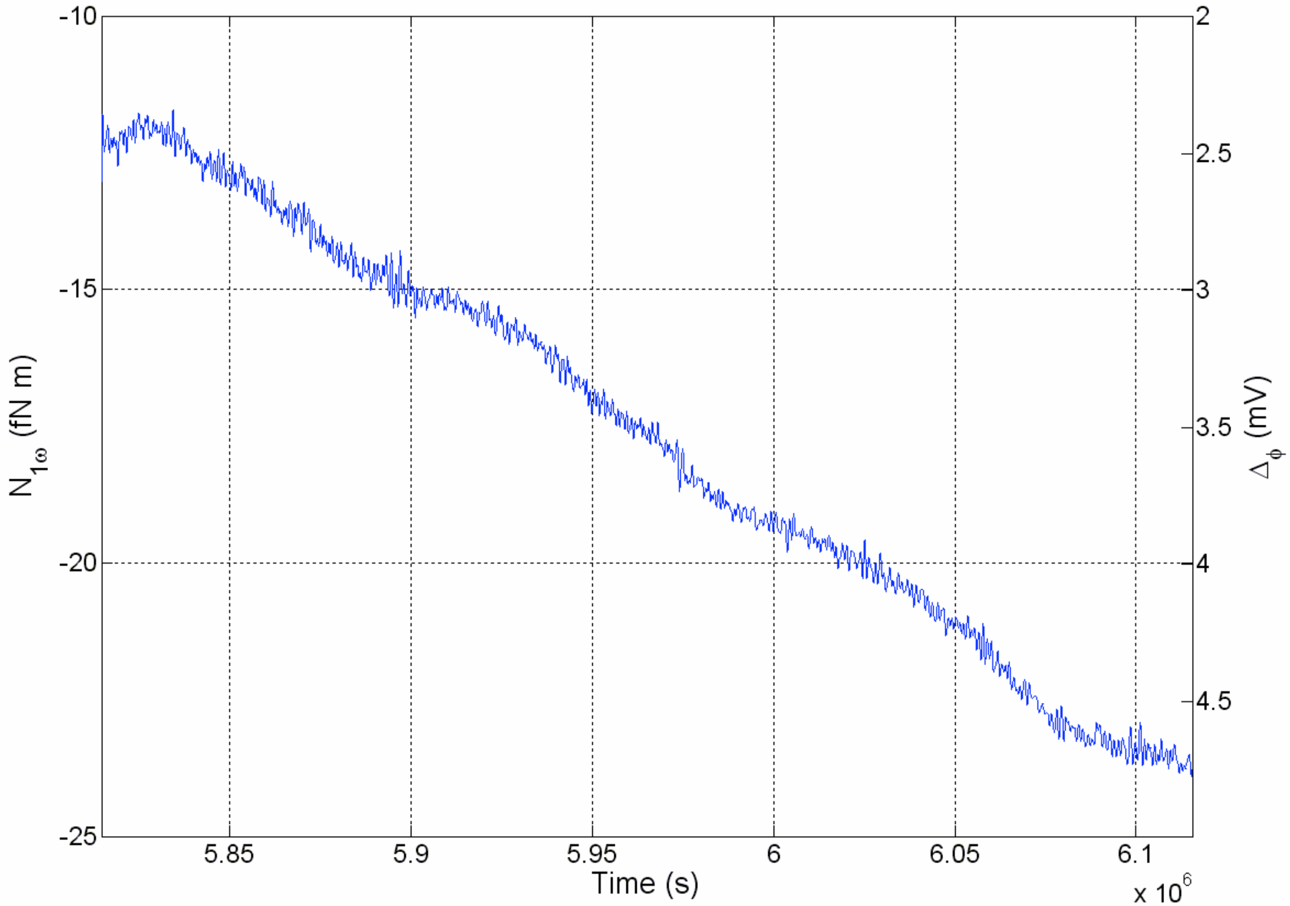
- Actuation amplifier noise (electronics)
- Thermal voltage fluctuations (δ)
- Drifting (not Brownian) DC bias $S_{\delta V}^{1/2}$

DC voltage difference: δV

- Test mass charge
- Residual unbalanced patch effects

LISA goal $v_{\mathrm{n}} \approx 20 \propto\mathrm{V/Hz}^{1/2}$ at 0.1 mHz

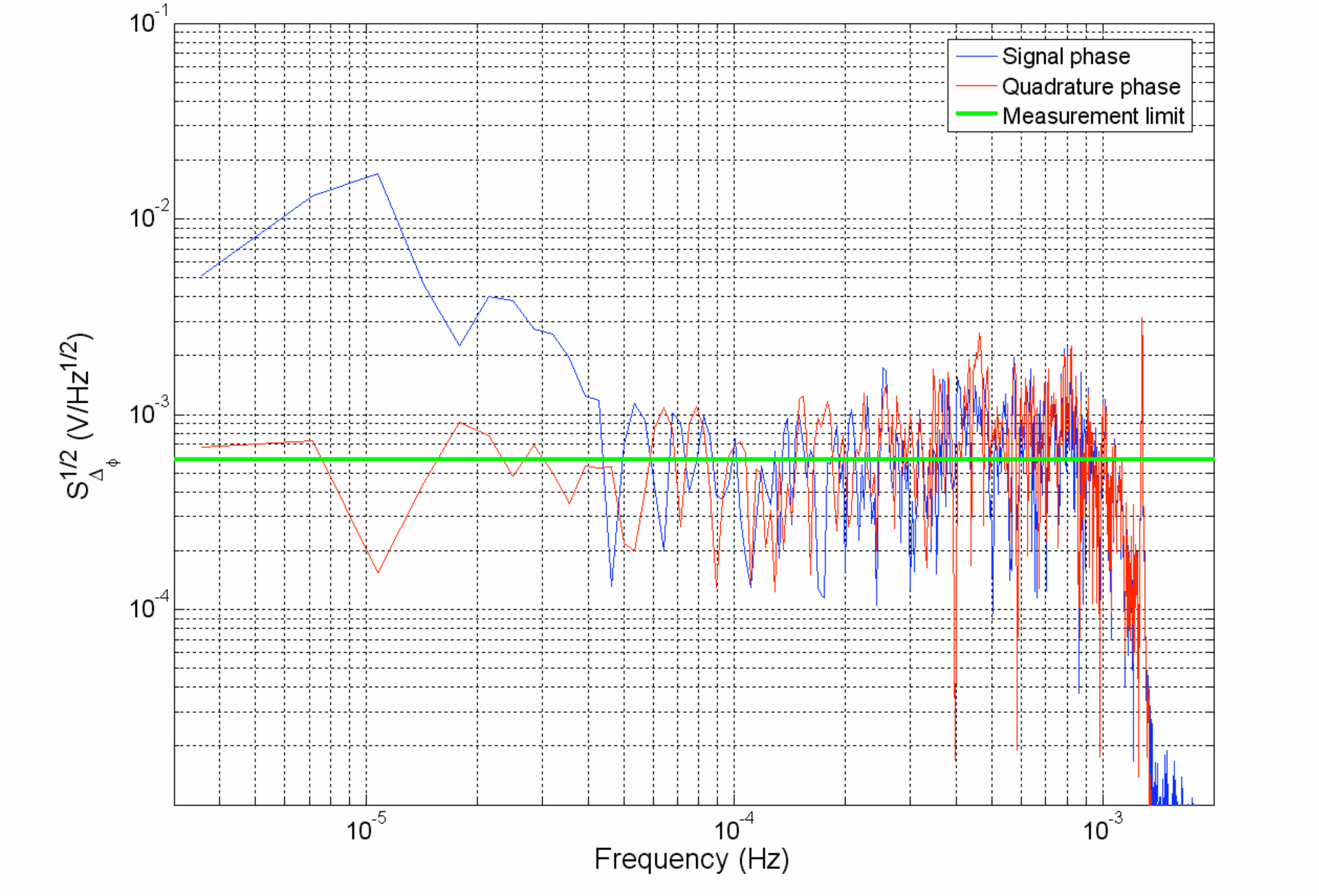
Stability in measured stray "DC" biases



$$\Delta_\phi \equiv \frac{\sum_i \frac{\partial C_i}{\partial \phi}\,\delta V_i}{\left|\frac{\partial C_x}{\partial \phi}\right|}$$

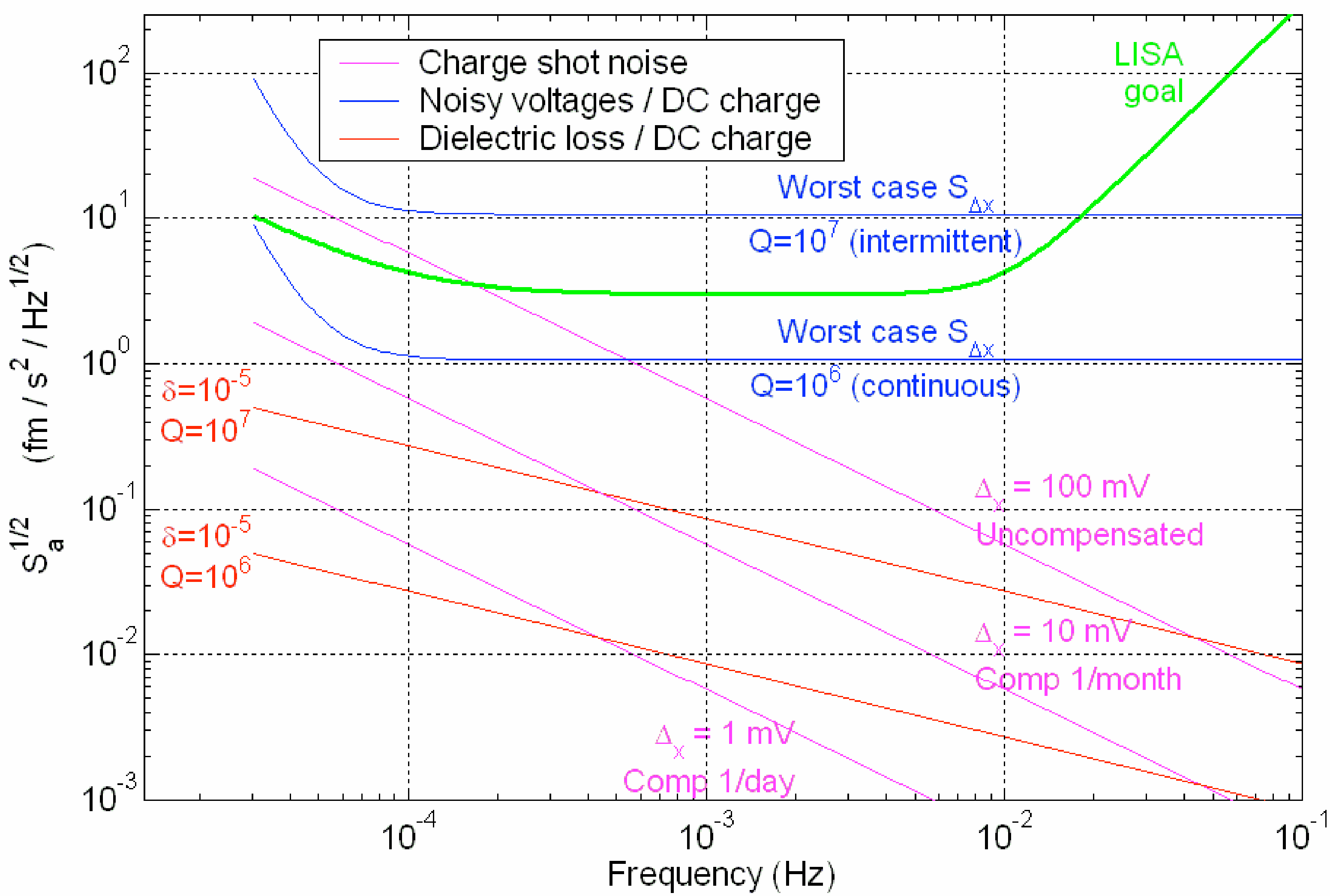
- Rotational DC bias imbalance Δ_ϕ measured over several days
- "DC" biases drift away away from (compensated) null over time
- Need to consider noise in "DC" biases

Measured noise in stray "DC" biases



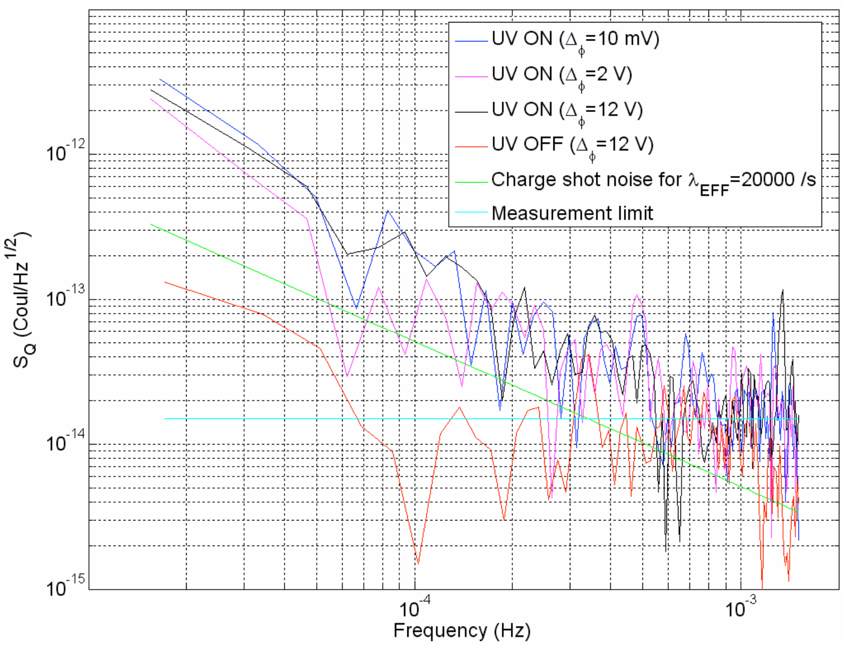
- Excess noise in Δ_ϕ observed below 50 ∝Hz
- Measurement limit (roughly 600 ∝V/Hz$^{1/2}$) factor 30 - 50 above LISA goal

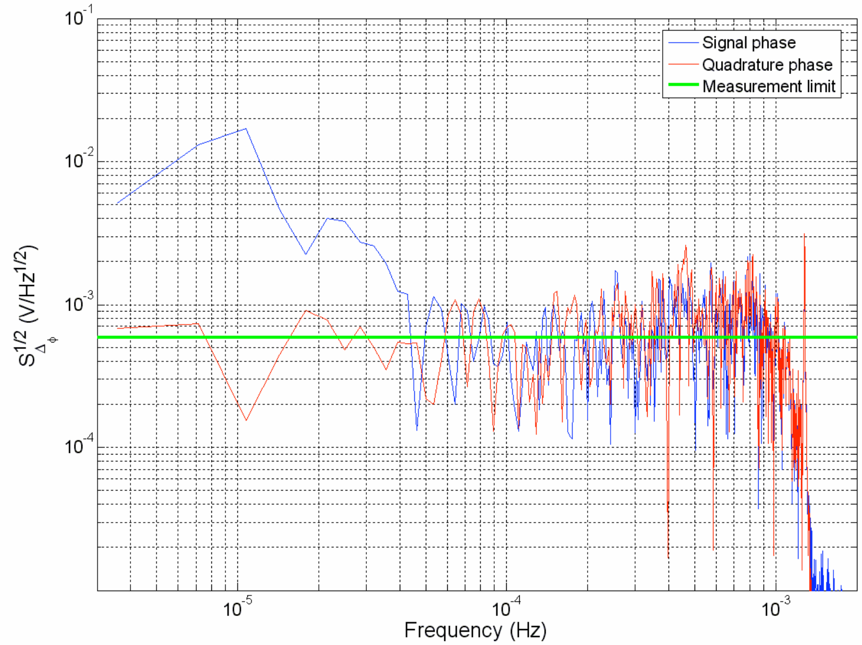
Noise budget for charge – stray voltage interaction



NB: "worst case" for stray voltage fluctuations is current measurement limit (true noise likely falls off with increasing frequency)

In-flight continuous measurement and compensation of Q, Δ_x





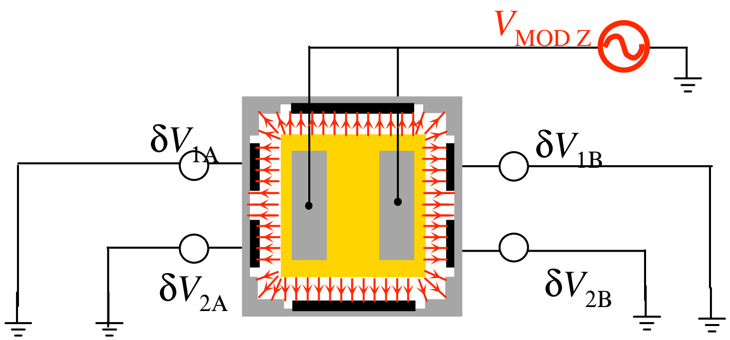
Continuous charge measurement

- Sufficient to see charge fluctuations below 0.1 mHz
- Allow “closed loop” continuous charge control to maintain $Q_{TM} < 10^{-6}$
- No disturbance on interferometry axis

Continuous measurement of Δ_x

- Sufficient to measure and compensate low frequency charge fluctuations
- Maintain low Δ_x, reduce low frequency $S_{\Delta x}$
- Demands a force signal on critical interferometry axis

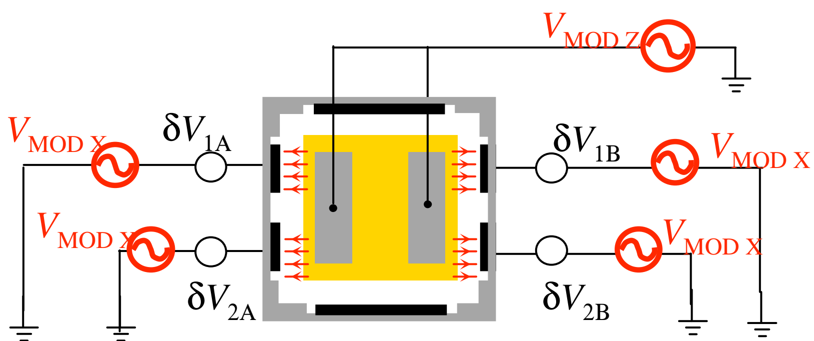
Different applied modulated E-fields
→ Distinguishing DC bias contributions



Modulated ΔV between TM and whole sensor

→ sensitive to sum of all DC biases, (as with TM charge)

$$N = -V_M \left[\sum \frac{\partial C_i}{\partial \phi} V_i \right] \equiv -V_M \left| \frac{\partial C_x}{\partial \phi} \right| \Delta_\phi$$



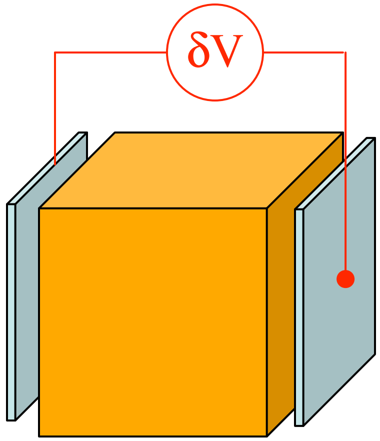
Modulated ΔV only between TM and x-electrodes

→ sensitive only to x-electrode DC biases

$$N = -V_M \left[\sum_{i(x\,\mathrm{el})} \frac{\partial C_i}{\partial \phi} V_i \right] \equiv -V_M \left| \frac{\partial C_x}{\partial \phi} \right| \Delta_{\phi(x\,\mathrm{el})}$$

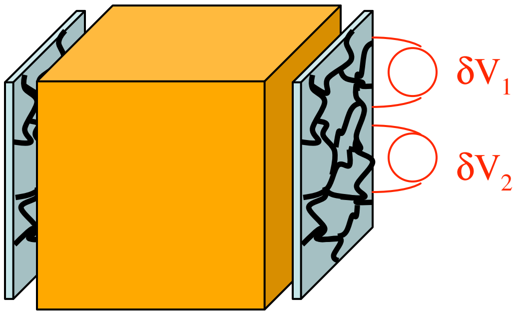
- Can distinguish and compensate DC bias contributions from different electrodes
- As DC biases arise in electrodes and guard ring surfaces, cannot simultaneously compensate both overall DC bias (Δ_ϕ or Δ_x) and individual electrode DC biases (δV_i)
- **True intrinsic DC bias values are important**

Effects of true, spatially inhomogeneous DC biases



Homogeneous DC biases

- Balancing δV eliminates coupling to charge and fluctuations in δV



True electrostatic potential distribution

- Balancing average δV eliminates coupling to TM charge
- **Coupling to individual domain voltage fluctuations cannot be compensated**

$$S_F^{1/2} = \sqrt{\sum_i \left(\frac{\partial C_i}{\partial x}\right)^2 \delta V_i^{\,2} S_{\delta V_i}}$$

$$\left\langle S_F^{1/2} \right\rangle \approx \sqrt{\frac{N}{4}} S_{\Delta_x}^{1/2} \sqrt{\left\langle \Delta_x^2 \right\rangle} \left| \frac{\partial C_x}{\partial x} \right|$$

[N = # domains / electrode]

- Not much data, model dependent!
- Could be worse than $Q_{TM} * S_{\Delta x}^{1/2}$ by a factor of several

Stray DC biases: conclusions

Measurements suggest:

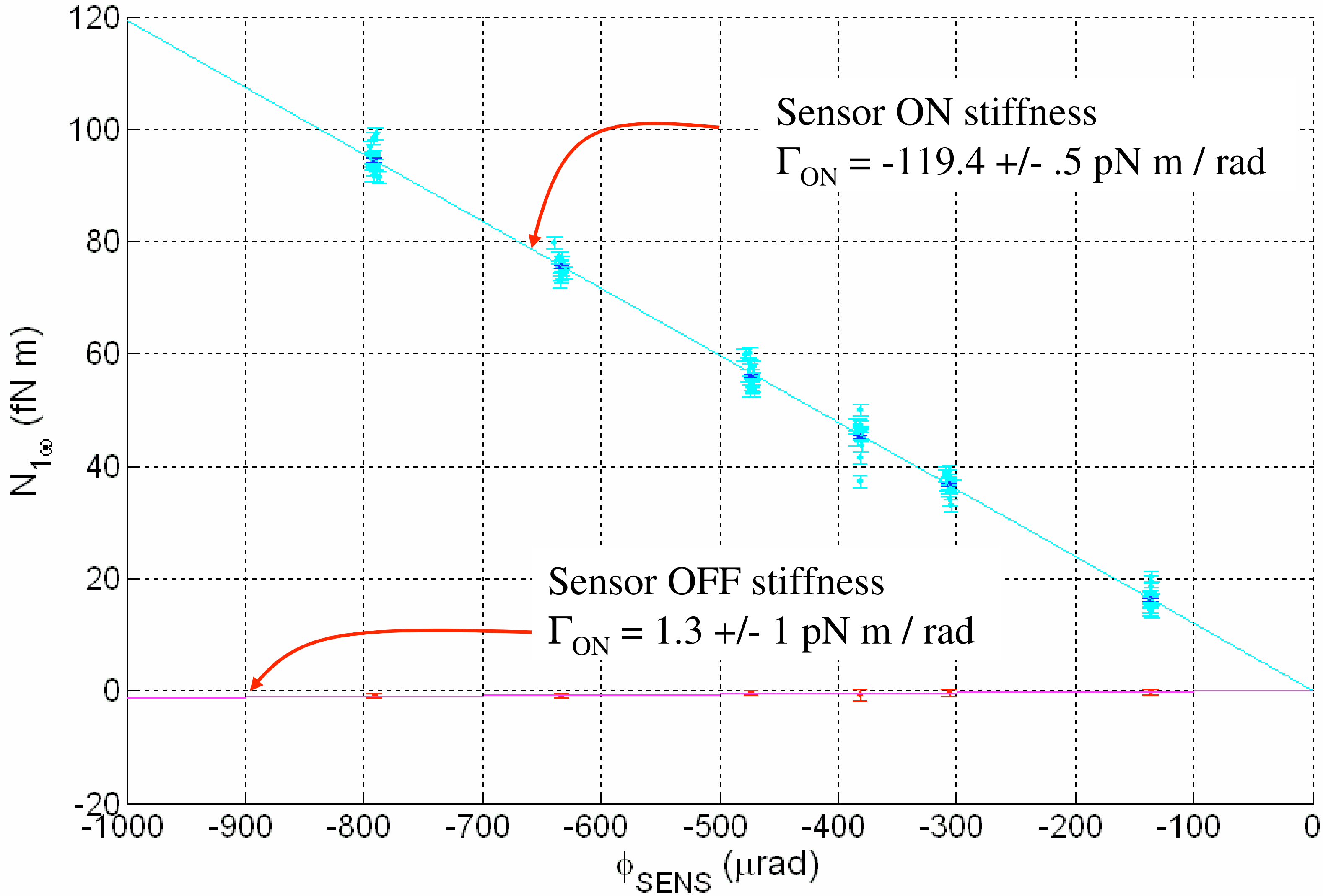
- Integrated average DC bias imbalances (Δ_x) of 100 mV
- Stiffness not likely to be an issue
- Compensation of (Δ_x) to < mV
- Low frequency drift / fluctuations
    - Need to correct periodically (or continuously) DC bias compensation
    - For f > 0.1 mHz → no excess noise in $S_{\Delta X}$ observed, but measurement limit well above LISA goal
    - For f< 50 ∝Hz, excess observed

- Interaction between TM charge and net DC bias threatens LISA acceleration goals (in worst case) only at lowest frequencies

    →Improved with continuous discharging

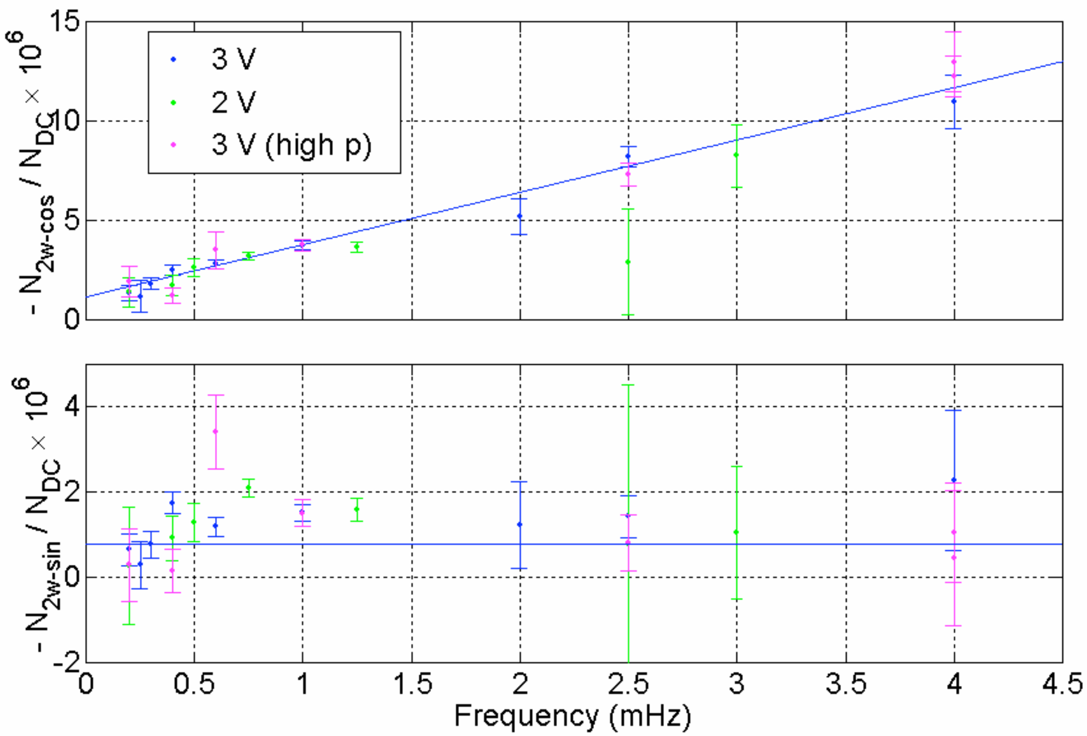
- Interaction between local DC biases and their own fluctuations a potential problem

LISA Symposium: Extra slides



- Stiffness from 100 kHz sensor bias roughly -121 pN m / rad
- Expected stiffness for 3.68 V amplitude bias = -179 pN m / rad (thus our model is off by 30%)
- Sensor off stiffness negligible

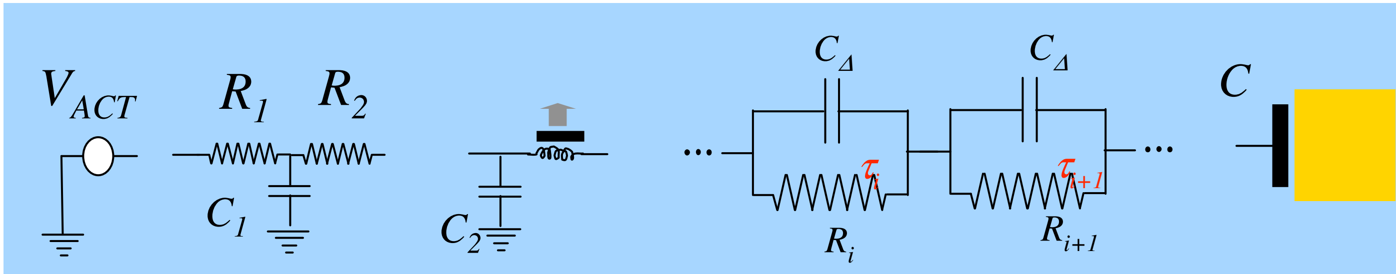
Dielectric Loss Angle Measurement Results



- 2ω cosine torque frequency dependence → ohmic delay time $\tau \approx 0.3$ ms (agrees with calculated value)
- 2ω sine + cosine intercept values → $\delta \approx 10^{-6}$
- likely not a problem for LISA!!

Electrodes 2W/1E	Averaged sine data		Linear fitted cosine data		
	δ ($/10^{-6}$)	χ^2	τ (ms)	δ ($/10^{-6}$)	χ^2
3 V (p ≈ 5.e-8 mBar)	.79 ± .07	1.8	.33 ± .02	1.06 ± .16	.86
2 V (p ≈ 5.e-8 mBar)	1.08 ± .09	1.36	.23 ± .05	1.48 ± .31	1.27
3 V (p ≈ 4.e-5 mBar)	.73 ± .14	2.25	.36 ± .03	.60 ± .27	1.27

Electrostatic noise source: thermal voltage noise from dissipation



Characterize surface + circuit dissipation with a capacitive loss angle δ:

$$v_n = \sqrt{4k_B T \frac{\delta}{\omega C}}$$

Thermal voltage noise mixing with DC voltages to produce force noise

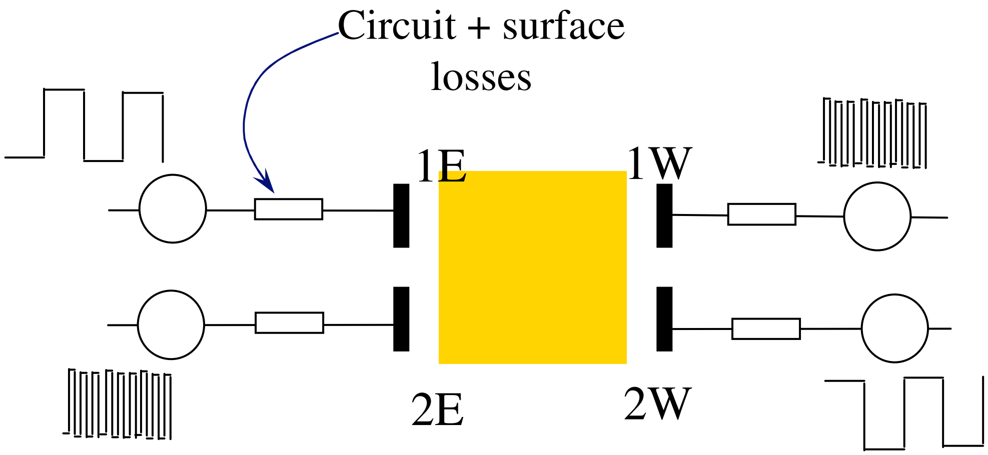
Equivalent

Thermal force noise generated by electrostatic dissipation (imaginary spring constant)

$$S_a^{1/2}(f) \sim .3\times10^{-15}\ \mathrm{fm/s^2}/\sqrt{\mathrm{Hz}}\left(\frac{\delta}{10^{-5}}\right)^{1/2}\left(\frac{10^{-4}\ \mathrm{Hz}}{f}\right)^{1/2}\left(\frac{Q_M}{10^7\ \mathrm{e}}\right)$$

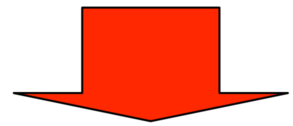
LISA requires δ < 10^{-5}

New technique to measure δ



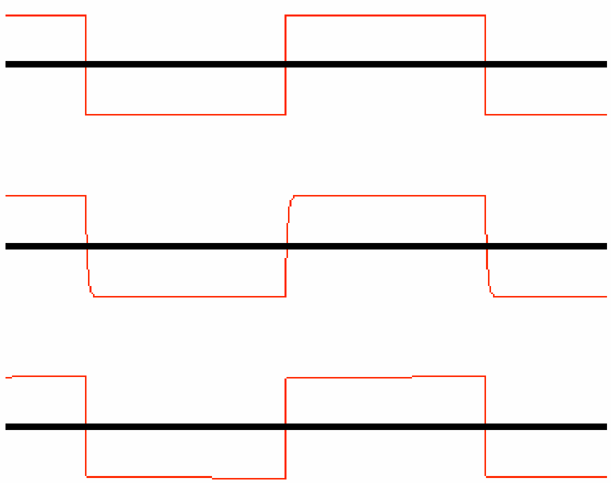
Force (torque) quadratic in voltage

$$F \propto V^2$$



perfect square wave voltage produces only DC force (torque)

Electrode voltage:

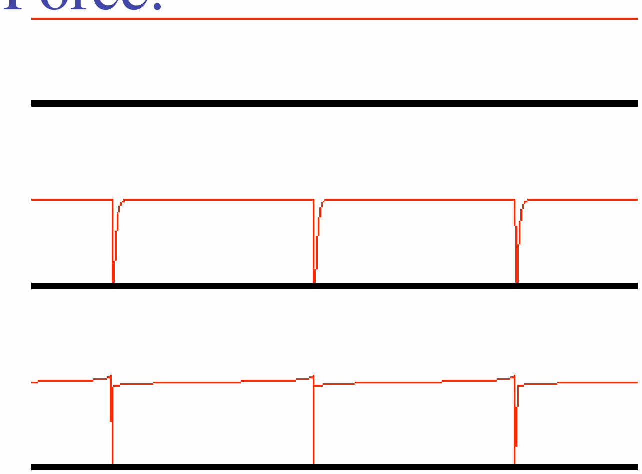


No losses

Ohmic delay

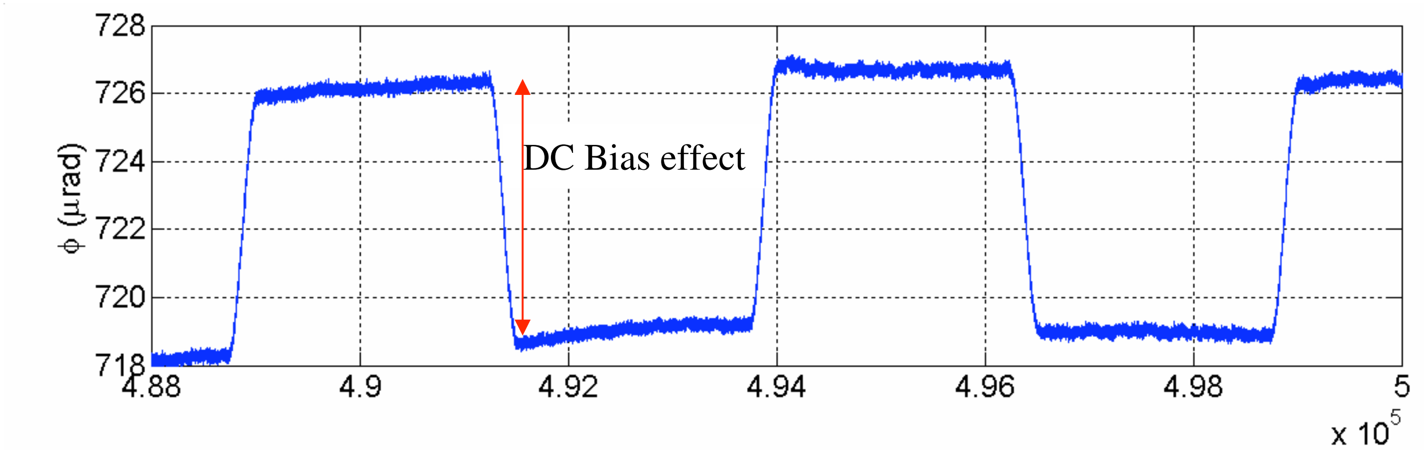
δ constant

Force:

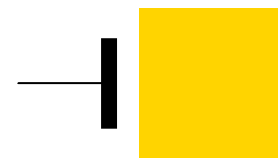


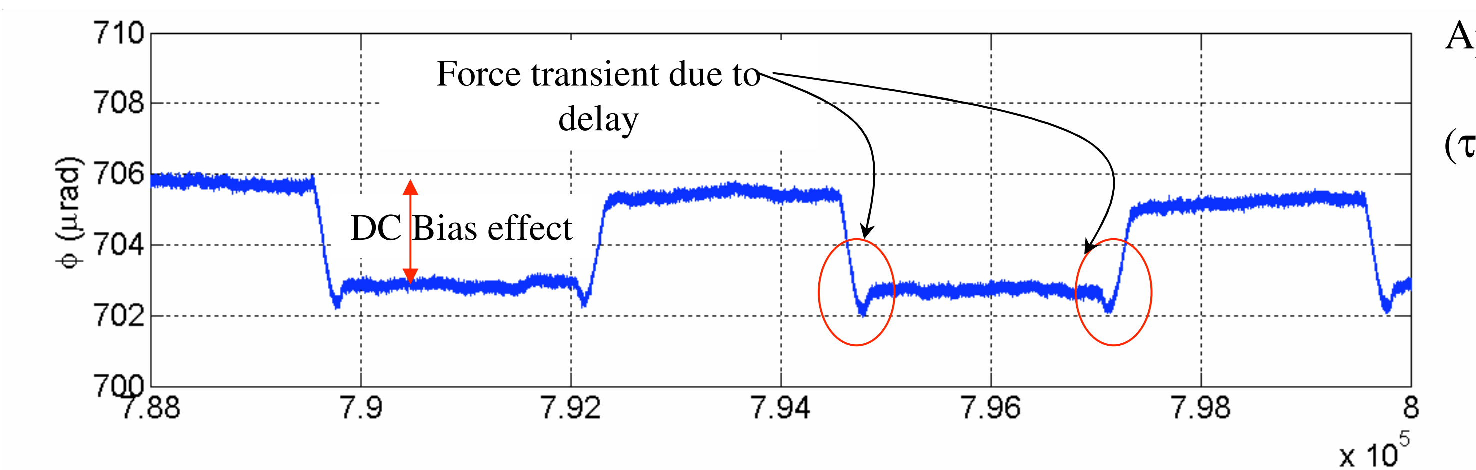
Measurement of dielectric losses: new direct measurement technique

Application of perfect square wave yields constant force
Any lossy element creates delays and thus force transients

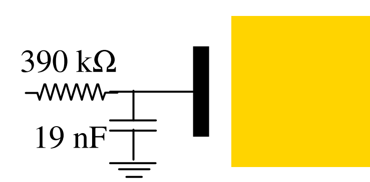


Direct application
(f = .4 mHz)

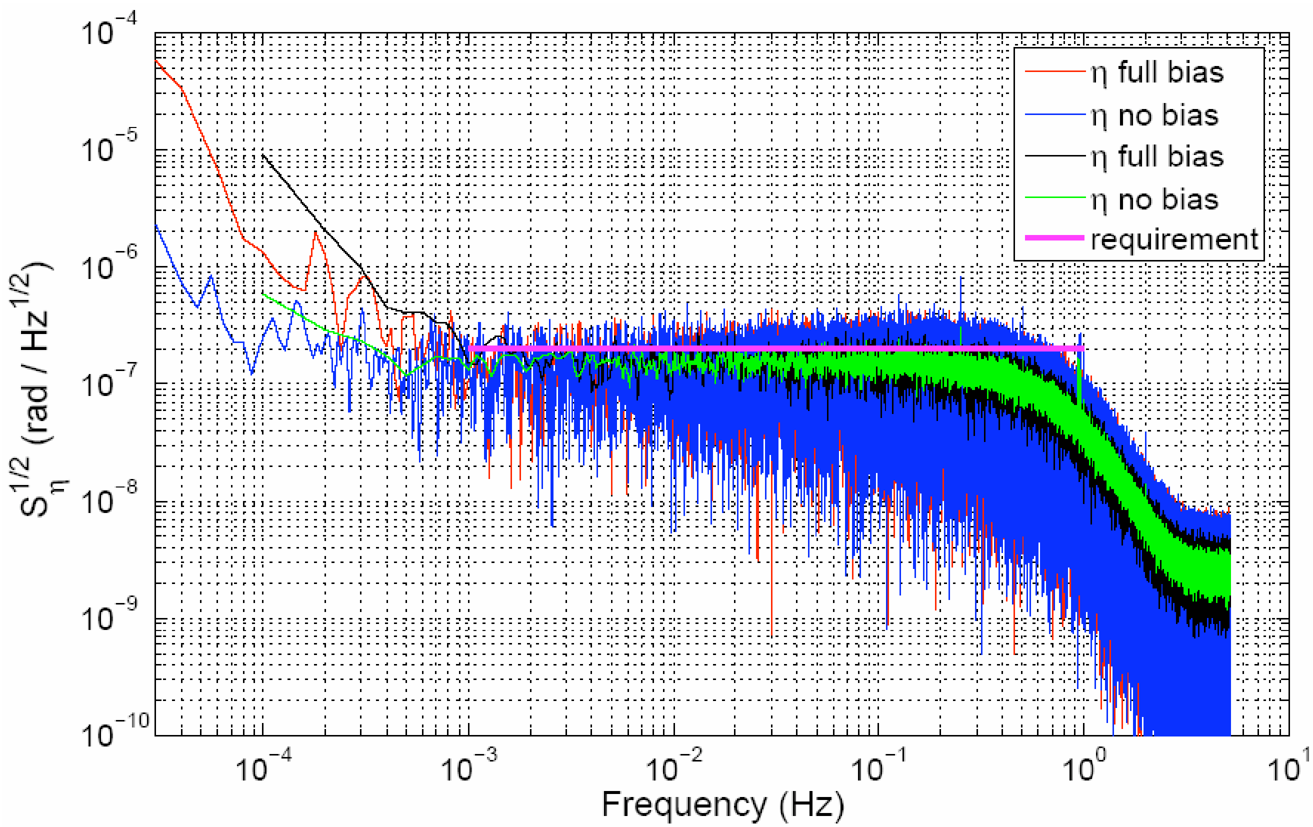




Application through an ohmic delay
($\tau \approx 7$ ms, $\delta \approx 2\ 10^{-5}$)



Sensor Displacement Noise Tests
Rigidly suspended TM

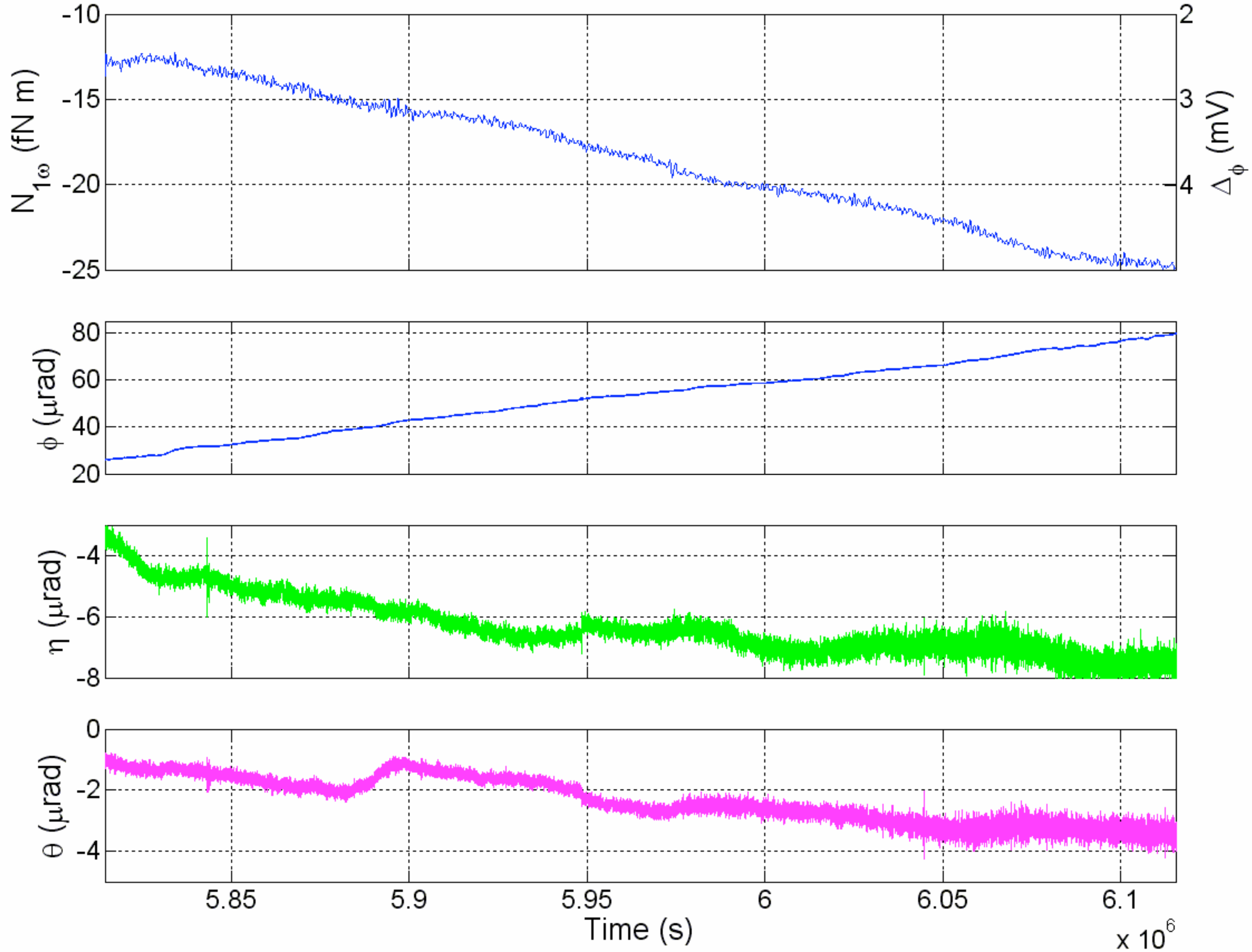


- Preliminary rotational noise measurements show compliance at 1 mHz

  → sensor works (with cables, small capacitances ecc)

  → we are able to prove it (mechanical suspension is stable enough)

DC Bias measurement fluctuation correlations with TM motion



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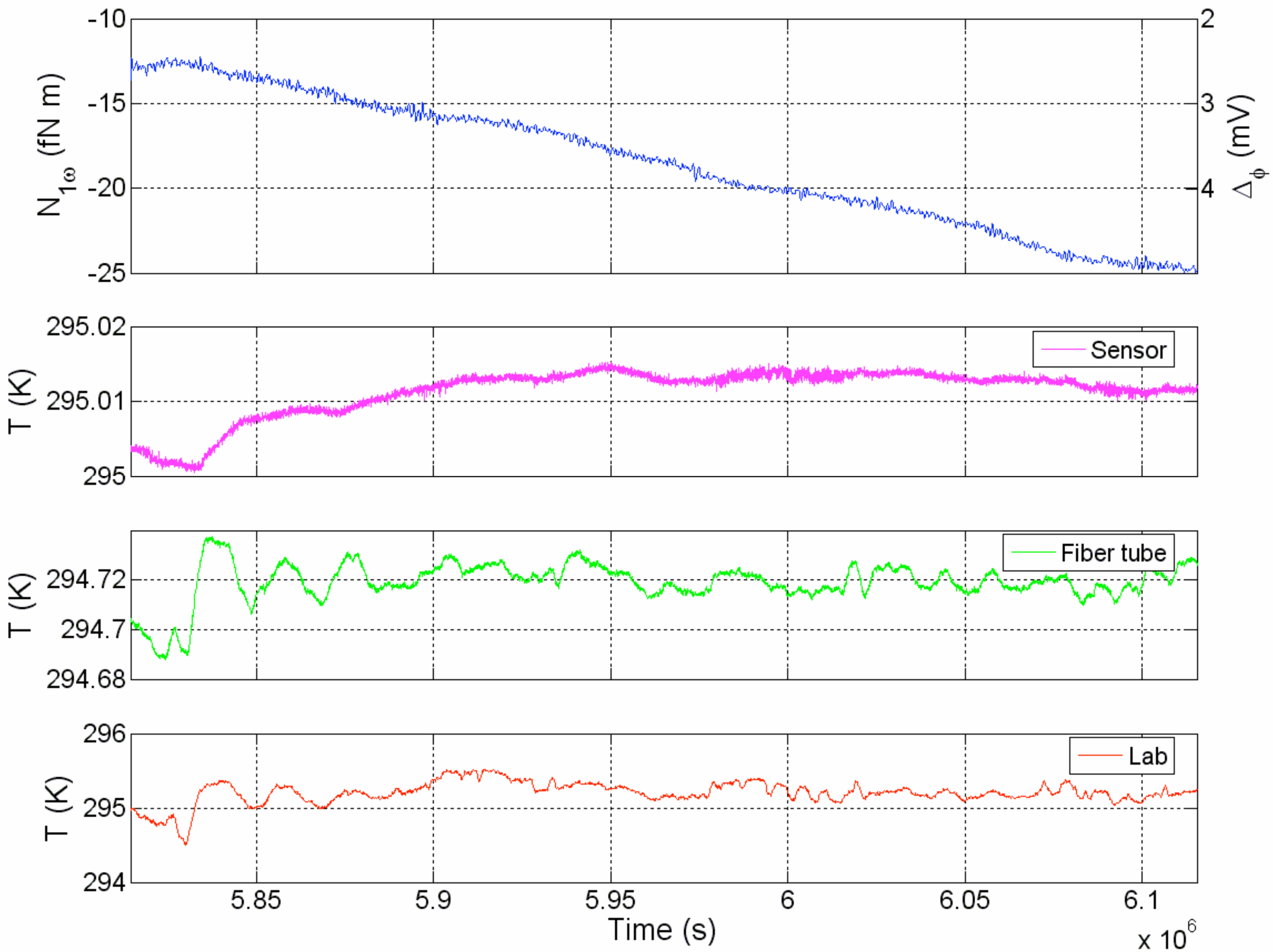
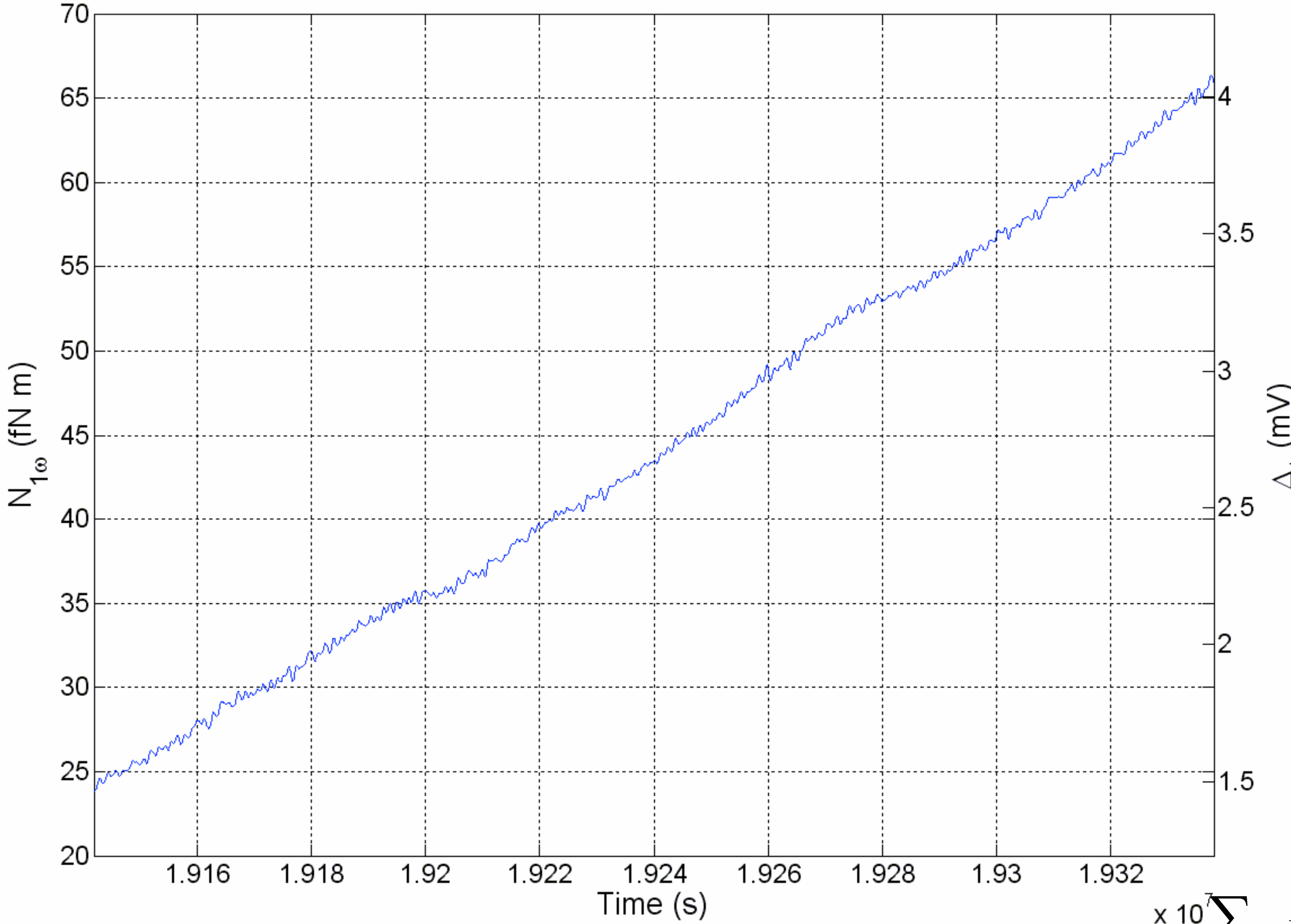


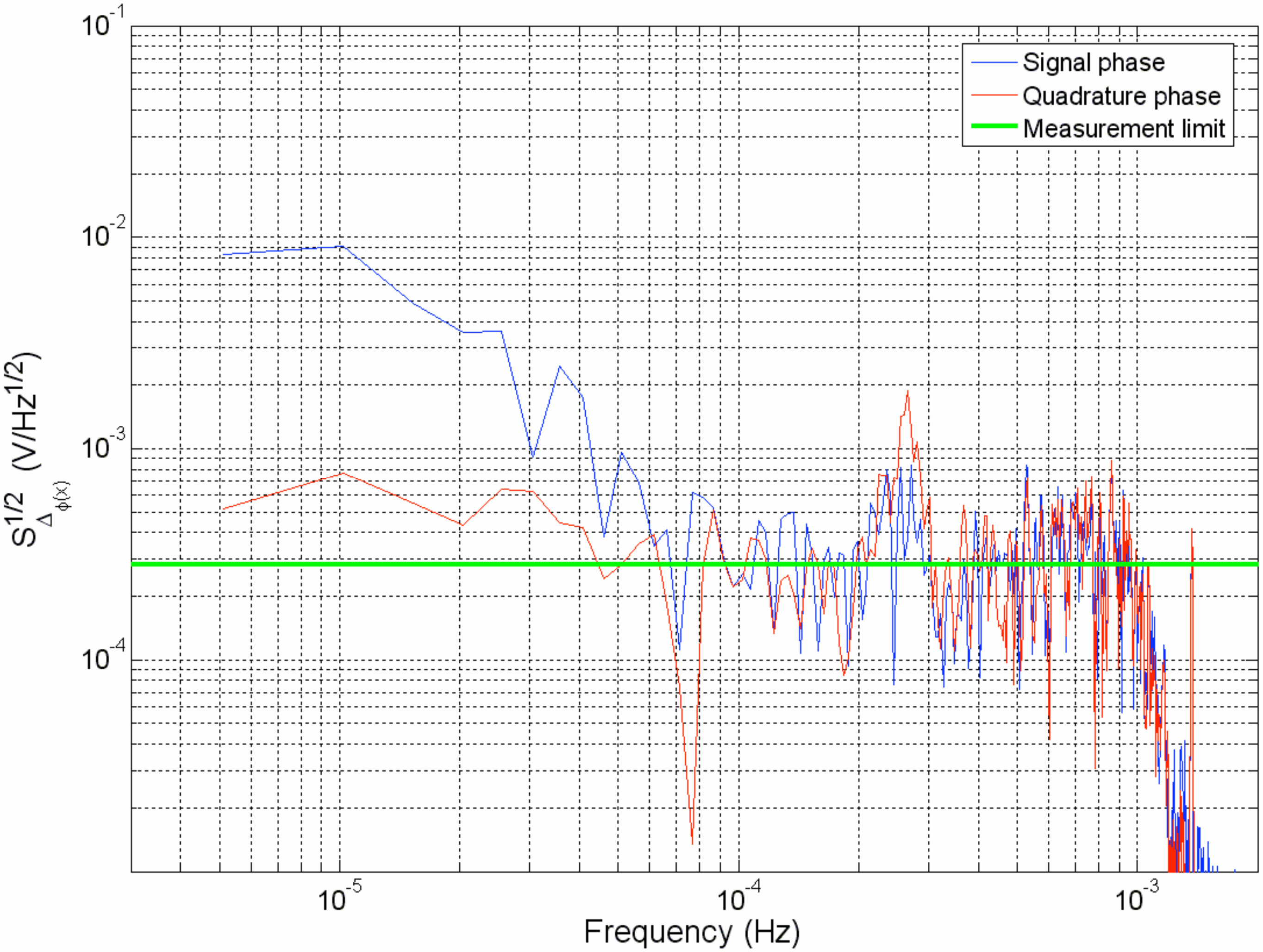
Stability of x-electrode DC biases



$$\Delta_{\phi(x)} \equiv \frac{\sum\limits_{x\,\text{electrodes}} \frac{\partial C_i}{\partial \phi}\,\delta V_i}{\left|\frac{\partial C_x}{\partial \phi}\right|}$$

Measurement of $\Delta_{\phi(x)}$ using V_{COMP}= + 20 mV

Noise in x-electrode DC biases



Measurement of $\Delta_{\phi(x)}$ using V_{COMP}= + 20 mV